UAGS Problem Set 5

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Problem 1 An easy way to show that $\mathbb{A}^2 \setminus (0,0)$ isn't affine: note that the inclusion into $\mathbb{A}^2$ induces an isomorphism of rings, but isn't an isomorphism.

Problem 2 Let X be a subset of $\mathbb{A}^n$ and let $k[X]$ be the ring of regular functions on X. Let I be an ideal of $k[X]$. Define

$$V_X(I) = \{p \in X \mid f(p) = 0 \text{ for all } f \in I\}.$$

Prove that the subsets of the form $V_X(I)$ are exactly the subsets of X that are closed in the subspace topology relative to the Zariski topology on $\mathbb{A}^n$.

Problem 3 Let $\phi : X \to Y$ be a regular map. For any $I \subseteq k[X]$, prove that

$$\overline{\phi(V_X(I))} = V_Y\left((\phi^*)^{-1} I\right).$$

(We did this at the end of class on Friday). If ϕ is an isomorphism with inverse ψ, prove further that

$$\phi(V_X(I)) = V_Y(\psi^* I)$$