UAGS Problem Set 4

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Problem 1 Which of the following conic sections are isomorphic to each other?

- Line
- Parabola
- Ellipse
- Hyperbola

Problem 2 By endowing any subset X of $\mathbb{A}^n$ with the subspace topology induced from the Zariski topology on $\mathbb{A}^n$, we can make sense of what it means for a function $f : X \to \mathbb{A}^1$ to be regular. Write $k[X]$ for the ring of regular functions on X. If $D(f)$ is the principal affine open associated to f, prove that $k[D(f)] = k[\mathbb{A}^n][f^{-1}]$.

Problem 3 We can similarly generalize the notion of a regular map and regular isomorphism to more general subsets of $\mathbb{A}^n$. Say that a subset of $\mathbb{A}^n$ is affine if it is isomorphic to an affine variety. Prove that $D(f)$ is affine. [Hint: note that $\mathbb{A}^1 \setminus 0 \cong V(xy - 1)$]

Problem 4 Prove that regular maps are continuous with respect to the Zariski topology.