UAGS Problem Set 3

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Problem 1 Prove the following facts about V and $\mathcal{I}$ without using the Nullstellensatz:

Part a If I_1 and I_2 are ideals of $k[\mathbb{A}^n]$, then $I_1 \subseteq I_2$ implies $V(I_2) \subseteq V(I_1)$.

Part b If X_1 and X_2 are subsets of $\mathbb{A}^n$, then $X_1 \subseteq X_2$ implies $\mathcal{I}(X_2) \subseteq \mathcal{I}(X_1)$.

Part c For all ideals I of $k[\mathbb{A}^n]$, we have $I \subseteq \mathcal{I}(V(I))$.

Part d For all subsets X of $\mathbb{A}^n$, we have $X \subseteq V(\mathcal{I}(X))$

Part e For all ideals I of $k[\mathbb{A}^n]$, we have $V(\mathcal{I}(V(I))) = V(I)$.

Part f For all subsets X of $\mathbb{A}^n$, we have$\mathcal{I}(V(\mathcal{I}(X))) = \mathcal{I}(X)$.

Problem 2 Prove that furthermore for subsets X of $\mathbb{A}^n$, we have $V(\mathcal{I}(X)) = \overline{X}$, where $\overline{X}$ is the closure of X with respect to the Zariski topology.